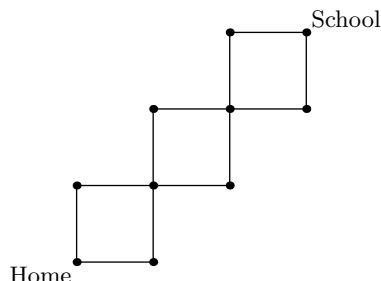
The background of the page is a complex, abstract pattern of thick, colored lines. The lines are primarily yellow, dark blue, red, and teal. They form various geometric shapes, including squares, rectangles, and zig-zags, creating a maze-like or circuit-like appearance. The lines are scattered across the page, with some forming larger, more prominent shapes like a large 'M' or 'W' in the center.

Team Round  
NC(SMC)<sup>2</sup>  
2022

## Team Round

**Instructions:** This round contains 10 problems and a time limit of 20 minutes, to be done in collaboration with your team. Have ONE team member submit all your answers here: <https://forms.gle/X8zY7xS3LrrtBCh37>. All answers are integers. Good Luck!

1. Alex is traveling from home to school along the roads shown below. Every minute, he either walks a segment up or to the right, until he arrives at school 6 minutes later. How many possible paths are there for Alex to go to school?



2. The letters N, C, S, M each represent a digit from 0 – 9 such that

$$\begin{array}{r}
 \phantom{+} \phantom{S} \phantom{M} \phantom{C} \\
 \phantom{+} \phantom{S} \phantom{M} \phantom{C} \\
 + \phantom{S} \phantom{M} \phantom{C} \\
 \hline
 \phantom{S} \phantom{M} \phantom{C} \\
 2 \phantom{0} 2 \phantom{0} 2
 \end{array}$$

The digits N and S are not zero. What is the 5 digit number NCSSM?

3. One of four siblings, Barry, Harry, Larry and Mary, stole a cookie from the cookie jar. Three of these siblings always lie, and one of them always tells the truth. When asked about who stole the cookie from the cookie jar, they gave the following responses:

- Barry: Larry stole the cookie from the cookie jar
- Harry: I stole the cookie from the cookie jar
- Larry: I did not steal the cookie from the cookie jar
- Mary: Barry did not steal the cookie from the cookie jar

Who stole the cookie from the cookie jar? Submit 1 if the answer is Barry, 2 if the answer is Harry, 3 if the answer is Larry, and 4 if the answer is Mary.

4. How many different ways are there to write 6 as a sum of  $n$  consecutive integers for some positive integer  $n$ ? For example,  $1 + 2 + 3 = 6$  is a way to write 6 as a sum of 3 consecutive integers.
5. How many ways are there to arrange the letters in the word POTATO such that all the O's come before all the T's?
6. Four circles are drawn in a plane, such that every circle intersects every other circle twice. How many regions of finite area do these four circles split the plane into?
7. The binary operation  $\spadesuit$  is defined as  $a \spadesuit b = ab - 3a - b + 6$ . For example,  $4 \spadesuit 5 = 4 \cdot 5 - 3 \cdot 4 - 5 + 6 = 9$ . What is the value of the below expression?

$$2022 \spadesuit (2021 \spadesuit (2020 \spadesuit (\dots (2 \spadesuit (1 \spadesuit 0)) \dots)))$$

8. If Sukrith, his enemy Rohan, and 6 others sit randomly in a line, what is the chance that they are sitting next to each other? If the probability is  $\frac{a}{b}$  for relatively prime positive integers  $a$  and  $b$ , submit the number  $a + b$ .
9. Isaac rolls three fair standard six-sided dice. The probability that there is at least one pair of dice whose top faces sum to at least 11 is  $\frac{a}{b}$ , where  $a$  and  $b$  are relatively prime positive integers. What is  $a + b$ ?
10. Find the number of ordered triples of integers  $(a, b, c)$  with  $1 \leq a, b, c \leq 10$  such that
- $a - b$  is a multiple of  $c$  (i.e.  $\frac{a-b}{c}$  is a integer)
  - $b - c$  is a multiple of  $a$
  - $c - a$  is a multiple of  $b$

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